<u>Octonions, the Three Flavours of Matter & a New Kind of Super-Symmetry</u>

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Abstract

It has been long theorised that there is some relationship between Octonions and the Standard Model. This paper reveals that there is just such a relationship and that it can solve many of the questions surrounding the Standard Model. These include: the seeming disorder of the particles and their masses; why there are three flavours of matter; and why some bosons are massless and some aren't. The solution comes from the precise ordering of the elementary particles in the Pentonions, before expansion via triangular numbers into the Octonions, leading to a potentially new kind of Super-Symmetry.

The Standard Model

The Standard Model was introduced in the 1970s, as a means of classifying and categorising the increasing number of particles that were being detected via experiment. At the time, physicists were running out of names for these particles and it was deemed necessary to put some order on what was being dubbed 'a particle zoo'. Physicists and theorists were able to achieve this by classifying the various particles by grouping particles into categories like leptons, quarks and gauge bosons and further classifying them by their charges, spins and masses. This led to the current standard model with its roughly 32 particles that we know and love today.

However, even with this, many physicists are not satisfied. It is true that the particle zoo has been put in order, but the order itself is not entirely regular and leaves us with many questions. Why do the particles have those particular masses? Why are there exactly three generations of quark? Why do some gauge particles have mass and others none? Why are 2nd and 3rd generation quarks unstable, while the corresponding leptons aren't? Why don't leptons and quarks interact directly? Why is there more matter in the Universe than anti-matter?

And so on…

Many attempts have been made to formulate answers to these questions. One of the most prevalent is Representation Theory. This theory was first proposed by Eugene Wigner, who noted an important relationship between Group Theory and the particles in the standard model.

But there is another less prevalent solution, which is given by the Octonions. To date, there have been many attempts to fit the particles of the Standard Model — along with their various spins and charges — into the division algebras that describe the octonions. There are numerous researchers, both amateur and professional investigating this possibility. One of the most high-profile examples is Nicole Fury's work, which relies on complicated mappings between the Octonions and Clifford Algebras.

There is something admittedly satisfying about attempting to create a table that brings order to the disparate elementary particles in the Standard Model. I myself have spend long hours attempting to do this, making use of both octonions and sedenions — without much success, I might add.

That was until, I looked at Δ and !Δ multiplication of the quaternions and things appeared to pop into place. It is now possible for me to describe nearly all of the particles in the Standard Model, with one simple table.

The method came to me, while I was eating a rather disgusting chicken and tomato ciabatta role in a Health Care Centre. I was pondering the submatrices of the 5-dimensional Higgs Boson. There are two 'sub-matrices'; the quaternions and the trionions. The trionions and pentonions are themselves sub-matrices of the octonions. Therefore the 3 matrices — using programming language — can be written as follows:

Pentonions = [1:5, 1:5]

Quaternions = [:4, :4]

Trionions = [1:4, 1:4]

There is clear and expected overlap here. My first attempt at structuring the particles into their respective sub-matrices can be seen in Fig. 1. This, as we can see, contains all of the particles of the Standard Model, excepting the W and Z bosons. We also see that there appear to be 8 Higgs Bosons. This is misleading however, as we shall see.

g	*d*	*c*	*t*	H^1
u	*y*	*ve*	*μ*	H^2
s	*e*	*y*	*vτ*	H^3
b	*vμ*	*τ*	*y*	H^4
H^1	H^2	H^3	H^4	*G*

Fig 1: Pentonion matrix 1

If we flip this table around by its anti-diagonal axis — in other words, we make the blue column the axis of multiplication — we see that wherever a 'H' and a 'H' multiply along the diagonal it forms a massless boson. In all other instances they make a massive particle. This is very similar to what we see in the Kronecker Delta, where a similar value by a similar value equals a '1' and otherwise equals a '0'. In this case it is the other way around; This is a traceless, or massless trace matrix. This not only makes it very similar to the traceless Hermitian matrices used in Gauge Theory; it is also suggestive of a Real numbered matrix, which is very promising.

The only value that is speculative is the Graviton, as we do not have experimental evidence for its existence. But the Graviton is theorised to have zero mass, so it definitely fits the pattern.

It is interesting that it should be the Higgs that determines the massless bosons. But when we consider that Fig. 1 is the embedding of an Octonion matrix, then it is clear that the massless bosons are the result of like imaginary, or real, terms multiplied together, just as it is in the DGO Standard Model.

There are many pluses to this table seen in Fig. 1. For instance, we see that all of the gluons and quarks are arranged in the quaternion group. The leptons, as we would expect, are arranged in the Trionion group and the Higgs and Graviton lies outside in the Pentonions.

The Three Flavours of Matter

You will recall, during the Quaternionic construction of the quarks and gluons and the Trionionic construction of the W/Z bosons and leptons, it was clear that there was no upper limit on the creation of these new particles. All we had to do was sum another particle with its companion and *hey presto*, we had another generation.

Now, we see that there is a limitation, by way of the shape of the dimensions of the matrix alone.

While this explanation might certainly appear simple — and it is — it was a long road getting to this point. The reason why the explanation is so simple, has to do with the simplicity of the question involved.

There is however possibly one other factor to consider.

Notice that there are three photons in this model, just as we noted in the Dionionic model. Earlier, we said that these three photons might represent different energy levels of light needed to generate the various flavours of matter. This suggests that once elementary particles reach a lower limit on their energy levels, they stop being able to form new flavours of particles, as you might expect and has been observed by experiment.

By extension, this implies that the gluon is somehow a photon that is even more energetic than gamma rays. Although this is far from certain.

Second Attempt

There are some clear problems with the table, however. For example, where are the W and Z bosons? And worse, why aren't the pentonions arranged in the normal pentonion matrix? They appear to be superimposed on the quaternions, which is not how we originally formulated them in [4], where they were the [:6, :6] sub-matrix of the Octonions.

This — I now realise — was a mistake, on my part. There are two ways to find a solution to this mistake. The first way, is to recalculate the Higgs particles based on the Pentonions in Fig. 4 to see if they still give us workable results. The other way is to rejig the table itself to include the correct version of the Pentonions.

Taking the second route first, we arrive at an entirely new table (Fig. 2). One of the upshots of this model is that it allows us to add the W and Z bosons into their correct positions.

This appears to work. But, we are left with what to do with the blue-cells and the Higgs bosons. The solution, I came up with is the one you see in Fig. 2. This creates a kind of chequerboard pattern featuring our 4 Higgs bosons and 8 Gravitons. Now, we really do have an ordered table of all the most important particles in the Standard Model that perfectly reflects the symmetries of the DGO model.

g	*d*	*1W1Z*	*c*	*WZ*	*t*
u	*y*	*ve*	*μ*	H^1	*G*
1W1Z	*e*	*y*	*vτ*	*G*	H^2
s	*vμ*	*τ*	*y*	H^3	*G*
WZ	H^1	*G*	H^3	*G*	H^4
b	*G*	H^2	*G*	H^4	*G*

Fig 2. Pentonion matrix 2

But there are several differences and corrections that we are forced to make. To begin with, as anyone will tell you, this is no longer a 5-dimensional matrix. It is clearly a 6-dimensional one. This implies that the Pentonions were actually Sextenions (?) all this time. But then that would imply that the Trionions are really quaternions, which is true in some sense, making the quaternions what exactly?

It is clear how the confusion arose.

The Trionions are made up of imaginary numbers, while the quaternions include a real number. The extension of the quaternions in Fig 6 are just the octonions; e_4 and e_5, which means that the rest of them are just the pentonions, as we defined them in [1]. So, technically, they are 5-dimensional, but taken altogether the whole system is 6-dimensional — much like parts of String Theory.

Luckily, this shift in dimension hasn't effected several of our key assumptions or inferences. For instance, the triune nature of our principle quark and lepton flavours still holds. It is also clear that a new set of W and Z bosons (an even lighter set) can exist where the gluon is. If so, this could be indicative of the union of the Strong and Weak force.

If anything, it looks like the Graviton is a less energetic form of the photon. But that can't be right, as the gravitons are now 5-dimensional and higher dimensions always have more energy. So, the strong-electroweak unification must wrap around to the Graviton to form the unification of the electroweak -strong force and gravity. If so, then it only relies on two of the Gravitons i.e. the two gravitons on the trace.

Which leads us to the next set question. Why are there 8 Gravitons? Moreover, if only the trace of the matrix is massless, and the Graviton itself is known to be massless, then why do six of them lie off the trace?

The 8 Gravitons

If we substitute the eight Gravitons for the eight colour charges of the gluons, we begin to see the pattern.

g	*d*	*1W1Z*	*c*	*WZ*	*t*
u	*y*	*ve*	*μ*	H^1	g_1
1W1Z	*e*	*y*	*vτ*	g_4	H^2
s	*vμ*	*τ*	*y*	H^3	g_5
WZ	H^1	g_6	H^3	g_3	H^4
b	g_2	H^2	g_7	H^4	g_8

Fig 3. Gluon-graviton relationship

This shows that there is a clear relationship between the Graviton and gluon colour charge. This relationship we might well expect, as the Graviton would need access to all four fundamental forces in order to interact with the various forms of matter.

The two linearly dependent gluons; g_3 and g_8 (or their graviton corollaries G_3 and G_8) lie on the trace, but none of the others do, which either suggests that those Gravitons do possess some mass or that they inherit their massless property from the trace.

g	*d*	*1W1Z*	*c*	*WZ*	*t*
u	*y*	*ve*	*μ*	H^1	G_1
1W1Z	*e*	*y*	*vτ*	G_4	H^2
s	*vμ*	*τ*	*y*	H^3	G_5
WZ	H^1	G_6	H^3	G_3	H^4
b	G_2	H^2	G_7	H^4	G_8

Fig 4. Pentonion matrix 2

This also tells us that the G_3 and G_8 may be crucial to the unification process. Perhaps, they are the linear summation of the traces of the electromagnetic U(1) and weak force SU(2) and SU(3) colour charges in some way? Whatever the truth, the relationship between the Graviton and the eight flavours of gluon tells us something important about the structure of the 5D graviton itself.

1	*i*	*j*	*k*	*E*	*I*
i	-1	*k*	*-j*	*I*	*-E*
j	*-k*	-1	*i*	*j*	*K*
k	*j*	*-i*	-1	*K*	*-j*
E	*-I*	*-j*	*-K*	-1	*i*
I	*E*	*-K*	*j*	*-i*	-1

Fig 5. This is the corresponding section of the Octonions, for comparison with Fig. 4.

Recall that all 8 of the gluon colour charges are part of a single 4-dimensional rhombic-dodecahedron (or hypercube). This suggests that all eight gravitons are similarly just different aspects of a single 5-dimensional polyhedron. From this perspective, there is likely only one generation of Graviton and by extension one flavour of Higgs. As we know, or suspect, the gluon (g_3 and g_8 included) is both massless and its own anti-particle. Therefore, all of the Gravitons inherit their 'lack of mass' from the massless trace Gravitons, which makes sense, if they are all one massless object.

However, the four Higgs particles (H^1, H^2, H^3, H^4) could refer to positive and negative charges of two flavoured particles, in which case there are two generations of Higgs and Graviton particles. A quick comparison between our DGO Standard Model and the corresponding section of the Octonions reveals that there are between 6 or even 8 Higgs variations. Correspondences like these are fun to play around with and could in future lead to instructive relationships between the pentonions and the particles, as well as the relationships between the elementary particles and one another. There may even be a potential for a relationship between octonion multiplication and particle interactions, but this remains to be seen.

Nevertheless, we can draw the following relationships:

I and -I = H_1, t and b

j and -j = μ, $v\mu$, G_4, G_5, G_6, G_7 and One W and One Z bosons

This suggests that there is a relationship between the Higgs and the top and bottom quarks. This relationship has been noted in many other places [5][6][7], particularly as it pertains to the top quark and the Higgs. As for the second group of particles, it looks like μ and vμ may transform into the G_4 to G_7 antiparticles via the One W and One Z bosons, given enough energy.

Charges, Spins and Masses

So far, we have been able to fit the particles of the Standard Model into the 6 x 6 Pentonions sub-matrix of the Octonions. This gives us a total of 34 particles (48 if you include all of the other Gravitons and Higgs particles as particles in and of themselves) and includes the antiparticles. The reason for the increase is down to the inclusion of One W and Z bosons.

We can take the analogies between the Pentonions and the particles further, by examining their relationship to the particle's various spins, charges and masses.

Plotting the particle spins does nothing, except confirm the same structure we had before, which I guess it indicative of being on the right track

(Fig. 6). Fig. 6 is obviously based on the assumption that the Graviton has spin-2, which I believe Ed Witten successfully proved to be the case.

1	1/2	1	1/2	1	1/2
1/2	1	1/2	1/2	0	2
1	1/2	1	1/2	2	0
1/2	1/2	1/2	1	0	2
1	0	2	0	2	0
1/2	2	0	2	0	2

Fig 6. Spin matrix

The mass matrix shows the same pattern again, more or less (Fig. 7.1). But, the Charge Matrix is a little different (Fig. 7). There is a regression line splitting the data into two sections; 21 and 15 datapoints in size, respectively. Points (1, 4) and (4, 1) no longer belong to the pentonion side and (3, 3) no longer fits into the trionion matrix.

0-1	1/3	0-1	2/3	0-1	2/3
2/3	0	0	1	0	0
0-1	1	0	0	0	0
1/3	0	1	0	0	0
0-1	0	0	0	0	0
1/3	0	0	0	0	0

Fig 7. Charge matrix with linear regression line showing how the data splits.

This same regression line can be applied to the Mass Matrix, as well. (Fig 7.1). These are interesting results and they call to mind the Pythagorean understanding that all square numbers can be generated by the sum of two neighbouring triangular numbers.

In this case, 15 + 21 = 36.

0.0	0.0047	50.0	1.28	91.19	173.1
0.0022	0.0	2.2e-08	0.10566	124.97	0.0
50.0	0.000511	0.0	0.0182	0.0	124.97
0.96	0.00017	1.7768	0.0	124.97	0.0
91.19	124.97	0.0	124.97	0.0	124.97
4.18	0.0	124.97	0.0	124.97	0.0

Fig 7.1: Mass matrix

36 is itself a triangular number, which suggests that the set of particles we have is one half of another set, which could either be 64 or 81 particles in size. We can, therefore, rearrange the masses into Pythagorean form in a 8x8 matrix (Fig. 8). The result shows a distinct asymmetry along the anti-diagonal. It looks like a group of Higgs Bosons are hovering around the top quark. Since the Higgs gives mass to the elementary particles, perhaps this grouping is why the top quark is so heavy. In fact, according to the Standard Model, this ‘mobbing’ of the t quark is precisely the reason for its unusually high mass.

0	0.0047	50	1.28	91.19	173.1	124.97	0
0.0022	0	2.2e-08	0.10566	124.97	124.97	124.97	--
50	0.000511	0	0.0182	0	124.97	--	--
0.96	0.00017	1.7768	0	0	--	--	--
91.19	124.97	0	0	--	--	--	--
4.18	0	0	--	--	--	--	--
0	124.97	--	--	--	--	--	--
124.97	--	--	--	--	--	--	--

Fig 8: Order 8 mass matrix using triangular numbers

The true asymmetry lies in the GeV triplets; 124.97, 173.1, 124.97 {at coordinates (0,4), (0,5), (1,4)} and 0, 4.18, 0 {at (5,0), (5,1) and (6,0)]}. But we can still see a sort of symmetry here in terms of respective magnitudes. The cells marked '- -' are as yet unknown quantities. But we can fill them in by creating mirroring the particles we already have into that region. This could lead to a new kind of Super-Symmetric Standard Model.

g	*d*	*1W1Z*	*c*	*WZ*	*t*	H_3	G_8
u	*y*	*νe*	*μ*	H^1	H^2	H^4	H^3
1W1Z	*e*	*y*	*ντ*	G_1	H^4	H^2	*t*
s	*νμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*
WZ	H^1	*y*	G_3	G_4	*ντ*	*μ*	*c*
b	G_6	G_7	*y*	*τ*	*y*	*νe*	*1W1Z*
G_2	H^2	G_6	H^1	*νμ*	*e*	*y*	*d*
H^3	G_2	*b*	*WZ*	*s*	*1W1Z*	*u*	g

Fig 9: A new kind of Super-Symmetry

Super Symmetric Octonions

As we might expect, this model is symmetric, along one axis. Along the other we see asymmetries, which once again show an imbalance of mass hovering around the top quark, and less around the bottom quark. Once again, this sheds light on the gross imbalance of mass apparent in the different quark flavours.

In the conventional Super-Symmetric model, the duplicate particles have heavier masses. In our version, the duplicate particles merely represent the anti-particles (or perhaps helicity). The anti-diagonal in Fig. 9 consists of particles and anti-particles, but since it is all composed of gauge bosons anyway, this isn't a problem. The same is true of gauge bosons that exist off this anti-trace line.

Now that we have a 8x8 matrix, we can easily associate these with the traditional octonions (Fig 10). This is not a concrete association, however, and I'm including it merely for the purpose of being completely thorough in our investigation. In order for these two systems to match, one of them needs to be flipped along its y-axis.

g	*d*	*1W1Z*	*c*	*WZ*	*t*	H^3	G_8
u	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3
1W1Z	*e*	*y*	*vτ*	G_1	H^4	H^2	*t*
s	*vμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*
WZ	H^1	*y*	G_3	G_4	*vτ*	*μ*	*c*
b	G_6	G_7	*y*	*τ*	*y*	*ve*	*1W1Z*
G_2	H^2	G_6	H^1	*vμ*	*e*	*y*	*d*
H^3	G_2	*b*	*WZ*	*s*	*1W1Z*	*u*	g

e7	e6	e5	e4	e3	e2	e1	1
e6	-e7	-e4	e5	-e2	e3	-1	e1
-e5	-e4	e7	e6	e1	-1	-e3	e2
-e4	e5	-e6	e7	-1	-e1	e2	e3
e3	e2	e1	-1	-e7	-e6	-e5	e4
e2	-e3	-1	-e1	e6	-e7	e4	e5
-e1	-1	e3	-e2	-e5	e4	e7	e6
-1	e1	-e2	-e3	e4	e5	-e6	e7

Fig 10: The Standard Model and the flipped Octonions

There are noticeable issues with the comparison. For starters, the top quark has is related to e2, but it has no corresponding -e2 value to represent its anti-particle. The same is true of the charm and down quark. It is possible that one of the other 480 possible octonion multiplication tables over the Reals would produce better results, but even if they did I doubt much of benefit would emerge from it.

g	*d*	*1W1Z*	*c*	*WZ*	*t*	H^3	G_8	- -
u	*y*	*νe*	*μ*	H^1	H^2	H^4	H^3	- -
1W1Z	*e*	*y*	*ντ*	G_1	H^4	H^2	*t*	- -
s	*νμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*	- -
WZ	H^1	*y*	G_3	G_4	*ντ*	*μ*	*c*	- -
b	G_6	G_7	*y*	*τ*	*y*	*νe*	*1W1Z*	- -
G_2	H^2	G_6	H^1	*νμ*	*e*	*y*	*d*	- -
H^3	G_2	*b*	*WZ*	*s*	*1W1Z*	*u*	g	- -
- -	- -	- -	- -	- -	- -	- -	- -	- -

$e_0 \mapsto 1$	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
e_1	-1	e_3	$-e_2$	e_5	$-e_4$	$-e_7$	e_6	e_9	$-e_8$	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$
e_2	$-e_3$	-1	e_1	e_6	e_7	$-e_4$	$-e_5$	e_{10}	e_{11}	$-e_8$	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}
e_3	e_2	$-e_1$	-1	e_7	$-e_6$	e_5	$-e_4$	e_{11}	$-e_{10}$	e_9	$-e_8$	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}
e_4	$-e_5$	$-e_6$	$-e_7$	-1	e_1	e_2	e_3	e_{12}	e_{13}	e_{14}	e_{15}	$-e_8$	$-e_9$	$-e_{10}$	$-e_{11}$
e_5	e_4	$-e_7$	e_6	$-e_1$	-1	$-e_3$	e_2	e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	$-e_8$	e_{11}	$-e_{10}$
e_6	e_7	e_4	$-e_5$	$-e_2$	e_3	-1	$-e_1$	e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	$-e_8$	e_9
e_7	$-e_6$	e_5	e_4	$-e_3$	$-e_2$	e_1	-1	e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	$-e_8$
e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_{12}$	$-e_{13}$	$-e_{14}$	$-e_{15}$	-1	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_9	e_8	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$	$-e_1$	-1	$-e_3$	e_2	$-e_5$	e_4	e_7	$-e_6$
e_{10}	e_{11}	e_8	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}	$-e_2$	e_3	-1	$-e_1$	$-e_6$	$-e_7$	e_4	e_5
e_{11}	$-e_{10}$	e_9	e_8	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}	$-e_3$	$-e_2$	e_1	-1	$-e_7$	e_6	$-e_5$	e_4
e_{12}	e_{13}	e_{14}	e_{15}	e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_4$	e_5	e_6	e_7	-1	$-e_1$	$-e_2$	$-e_3$
e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	e_8	e_{11}	$-e_{10}$	$-e_5$	$-e_4$	e_7	$-e_6$	e_1	-1	e_3	$-e_2$
e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	e_8	e_9	$-e_6$	$-e_7$	$-e_4$	e_5	e_2	$-e_3$	-1	e_1
e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	e_8	$-e_7$	e_6	$-e_5$	$-e_4$	e_3	e_2	$-e_1$	-1

Fig 11: The expanded Nononion matrix and the Sedenions

One reason for this is that we are only using the 8x8 matrix. It obviously makes more sense to use the 9x9 matrix. This would be a Nononion matrix (it is pronounced exactly how it is written; non-onion) and would thereby bring Sedenions into the mix. However, it would also lead us into a situation whereby many of the particles and their properties are completely unknown,. As for what these new particles might be, I would speculate that they are of the ‘Dark Matter’ variety. This method may gives us a means of inferring the properties of some of these particles based on the properties of the particles in the known Standard Model.

The Sedenions

Now, we are definitely getting somewhere. Here we have four copies of our reflected Standard Model, each corresponding to a Sedenion multiplication (Fig. 12). We’ve solved the particle anti-particle issue and there’s no need to flip the Sedenions. The top lefthand quadrant is the original matrix corresponding to the Octonions, the bottom righthand matrix is nearly the same. It differs in so much as all of the signs are inverted; allowing for the anti-particles (opposite signs in the same quadrant likely indicate helicity). But it is also made by the multiplication of purely Sedenionic numbers with themselves.

	e0	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12	e13	e14	e15
e0	g	d	1W	c	WZ	t	H^3	G_8	g	d	1W	c	WZ	t	H^3	G_8
e1	u	y	ve	μ	H^1	H^2	H^4	H^3	u	y	ve	μ	H^1	H^2	H^4	H^3
e2	1W	e	y	vτ	G_1	H^4	H^2	t	1W	e	y	vτ	G_1	H^4	H^2	t
e3	s	vμ	τ	G_4	G_5	G_1	H^1	WZ	s	vμ	τ	G_4	G_5	G_1	H^1	WZ
e4	WZ	H^1	y	G_3	G_4	vτ	μ	c	WZ	H^1	y	G_3	G_4	vτ	μ	c
e5	b	G_6	G_7	y	τ	y	ve	1W	b	G_6	G_7	y	τ	y	ve	1W
e6	G_2	H^2	G_6	H^1	vμ	e	y	d	G_2	H^2	G_6	H^1	vμ	e	y	d
e7	H^3	G_2	b	WZ	s	1W	u	g	H^3	G_2	b	WZ	s	1W	u	g
e8	g	d	1W	c	WZ	t	H^3	G_8	g	d	1W	c	WZ	t	H^3	G_8
e9	u	y	ve	μ	H^1	H^2	H^4	H^3	u	y	ve	μ	H^1	H^2	H^4	H^3
e10	1W	e	y	vτ	G_1	H^4	H^2	t	1W	e	y	vτ	G_1	H^4	H^2	t
e11	s	vμ	τ	G_4	G_5	G_1	H^1	WZ	s	vμ	τ	G_4	G_5	G_1	H^1	WZ
e12	WZ	H^1	y	G_3	G_4	vτ	μ	c	WZ	H^1	y	G_3	G_4	vτ	μ	c
e13	b	G_6	G_7	y	τ	y	ve	1W	b	G_6	G_7	y	τ	y	ve	1W
e14	G_2	H^2	G_6	H^1	vμ	e	y	d	G_2	H^2	G_6	H^1	vμ	e	y	d
e15	H^3	G_2	b	WZ	s	1W	u	g	H^3	G_2	b	WZ	s	1W	u	g

Fig 12: Eight copies of the Standard Model corresponding to the Sedenions

The two other quadrants represent the Dark Matter particles and anti-particles. They are the result of left and right multiplication of pure Octonions with Sedenions. Therefore, we could say — if the analogy holds — that dark matter is the result of Sedenion multiplication between matter and anti-matter. This means that it exists at 'right angles' to ordinary matter and may explain why it isn't seen to interact. Furthermore, anti-matter is actually the result of dark matter multiplied by dark anti-matter.

Now that we have this, let's see what we can do with it. We'll start with the tau (τ). The matter tau is: τ= (e3.e2), the anti-matter tau is τ* = (e11.e10), the dark matter tau is |τ| = (e11.e2) or (e3.e10), the dark matter anti-tau is |τ*| = (e5.e12) or (e13.e4). Now, let's look at what these numbers breakdown to:

e4 = WZ, e2 = 1W, e11 = s, e5 = b,
e13 = b, e10 = 1W, e3 = c

So what can be made from all of these? That's right. We can use these particles to make bottom quark decay diagrams.

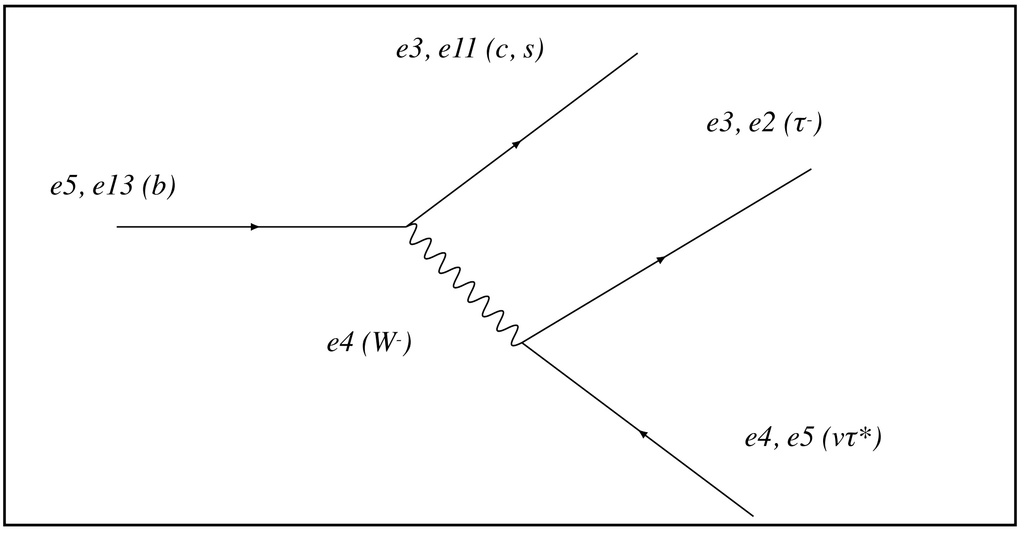

Fig. 13: Bottom quark decay diagram.

It is further interesting—although no doubt expected—that this reaction should begin with the tau and the anti-tau and then transition to the W boson and bottom quarks etc. There are probably many more such interactions lurking in the Sedenions, but how many are not right? Presumably, there are some. And where are the infinite number of interactions, we would expect from quantum physics? At a glance, they're not there.

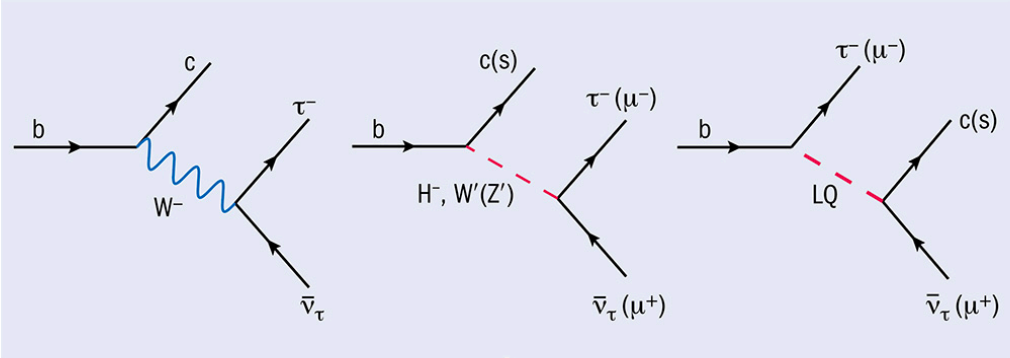

Fig. 14: More b quark decay diagrams. Image credit [8]

While the Sedenions may not provide all of the answers, as far as I'm concerned, this has been a satisfying investigation into the subject. Compared with other methods of fitting the subatomic particles into a kind of ordered symmetry using complex numbers, I think that this one provides the best results. Overall, this suggests the important relationship between these hypercomplex numbers with that of the Standard Model and particle physics.

Conclusion

All 32 particles of the Standard Model (along with several new ones) can be represented in the form of a 6x6 Pentonionic grid. This configuration explains why there are only three flavours of matter in existence. Massless bosons are shown to conform with the multiplication of like-valued imaginary numbers i.e. $i*i$=-1. The Pentonion grid also reveals an important relationship between the graviton and the 8 colour charges of the gluon and hints that the two linear dependent gluons might help with the unification of the fundamental forces. A deeper structure of the Pentonions is hinted at via triangular numbers. This structure is expanded into an 8x8 matrix, and finally a 16x16 Sedenion grid, leading to a new kind of symmetric structure of particles, antiparticles, as well as dark matter particles and antiparticles and helicity.

Citations

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