Octahedral Symmetry in Elementary Particle Physics

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Abstract

The importance of Octahedral Symmetry (O_h) to the results of the Dimensional Gate Operator (DGO) Standard Model is abundantly clear. Octahedral Symmetry links the quaternions to the various Lie Groups and Spin Groups and therefore shows the deep relationship between the DGO conception of the Standard Model and the more conventional approach.

1. Introduction

The DGO Standard Model contains various geometric figures including; cubes, rhombic dodecahedra, cuboctahedra, rhombicuboctahedra and chamfered cubes. These all belong to the Full Octahedral Symmetry Group O_h [1]. In fact, nearly every geometric form contained within the DGO Standard Model belongs to O_h. Among the exceptions are the icosahedron, which has chiral tetrahedral symmetry [2], but even this is a subset of O_h [3].

Full Octahedral Symmetry largely deals with the geometry of the cube, rather than the octahedron [1]. This is because the cube is the dual of the octahedron in the first place. The cube has 6 faces and 8 vertices. This means O_h has 48 elements, because we can have 6 rotations by 8 reflections making 48. All geometries that share O_h have this same feature.

O_h is related to S_4, which is the symmetry group of 24 elements. These 24 elements can be represented by rotations of the cube and then a further 24 reflections of each of these rotations to complete the O_h group of 48 [1].

The number 48 is an important number. It turns up when dealing with quaternions. Each quaternionic vector can be reinterpreted as a real numbered matrix of 48 elements (technically it is 48 sets of four real numbered matrices according to Wikipedia, but this doesn't detract from the truth of the previous statement). The similarities between O_h and the quaternions don't end there. In fact, it can be shown that the 48 elements of GL(2,3) (a Binary Octahedral Group) split into two symmetric groups of 24, which then become the two Binary Tetrahedral Groups, both of which equally describe the quaternionic vertices of the 4-dimensional 24-cell [4]. This is a fun coincidence, but probably not surprising, as the 24-cell has — after all — hyper-octahedral symmetry of dimension-4.

The number 48 is also to be found in the DGO Standard Model, where it represents the finally tally of particles in the model [5]. In the conventional Standard Model, we have 24 fermions and six bosons (the gluon, photon, the electro-weak bosons and the Higgs). The DGO is in full agreement with this, but also adds 4 variations of the Higgs and the hypothetical graviton (of which there are 8 variations — much like the gluon). But since, we are adding 8 variations of the graviton, we must also add the 8 variations of the gluon, from which they were derived. Together with the photon and the 3 electro-weak boson this gives us a total of 48.

24 fermions + 8 gluons + 8 gravitons + 4 Higgs + 3 weak bosons + 1 photon = 48

Given that symmetry groups are such a complex field, I would like to spend time investigating them in more depth. This paper represents the exploration of these groups and their apparent relationship to the particles of the DGO Standard Model.

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2. The Stone of the Philosophers

All of the elements of S_4 can be represented by the 24 vertices of the truncated octahedron, also known as the permutahedron. We can do the same for the 48 elements of O_h using the truncated cuboctahedron (CO). This would suggest that the truncated cuboctahedron is part of the geometric forms in the DGO Standard Model. If this is true, it has yet to be located. It is possible that it exists as the addition of the Higgs and the Graviton, or as some 6th-dimensional polyhedra existing above all the others. The closest object I have found so far is one created by the addition of a gluon and a down quark, but I have yet to identify what the object is.

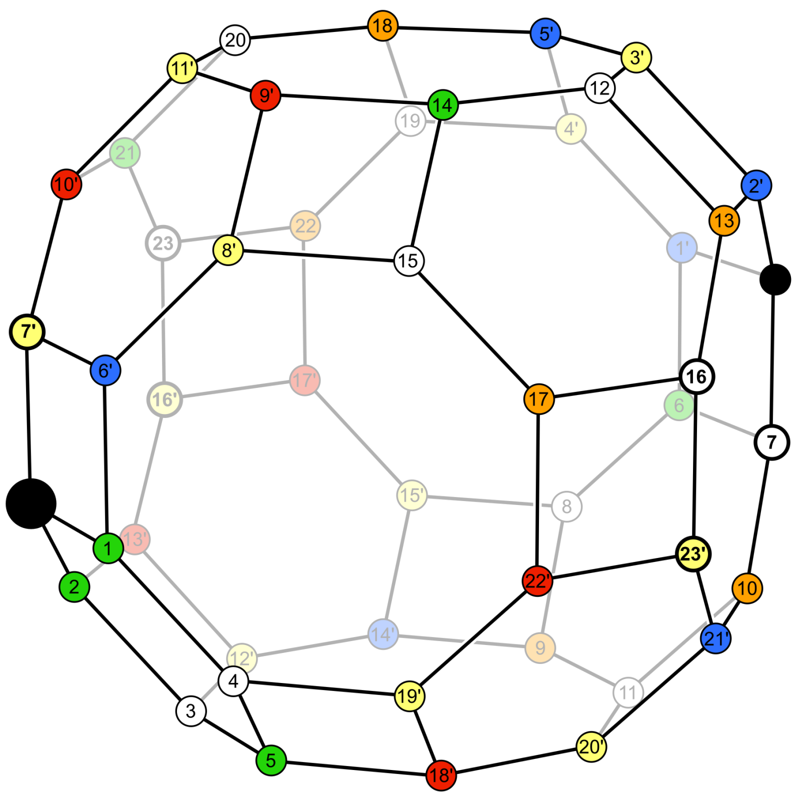

Figure 1: Octahedral symmetry represented by the truncated cuboctahedron [1, 7]

It is worth noting that there exist multiple subgroups of the the octahedral symmetry group seen in Figure 1, including; T_d, O, T_h, D4h, T, and etc. Each of these is further represented by a geometric figure (i.e. a polyhedra, a surface or a line; depending upon the size or dimensions of the group itself). For example, the subgroup O can be inscribed within the truncated cuboctahedron as the nonuniform snub cube. A further example would be the subgroup T_h, which forms a cantic snub octahedron. We actually saw this figure before, as the non-uniform rhombicuboctahedron, and as a result of the geometric quantisation of the excitations of an electron. It is the repeat of this figure, which makes me believe that the truncated cuboctahedron, as well as potentially the nonuniform snub cube will be found somewhere within the DGO Standard Model.

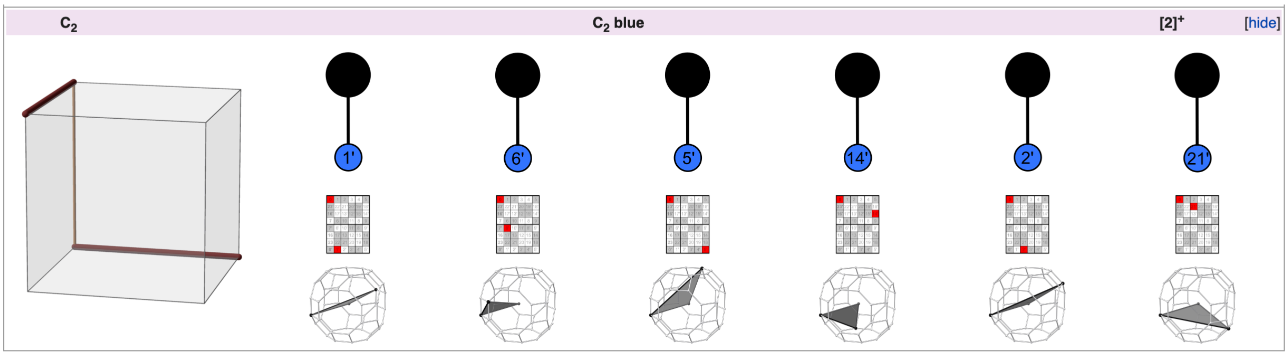

Figure 2: C_2 Blue Subgroup order 2 [1, 7]

The cantic snub octahedron, which in the limit became an icosahedron was the first geometric figure to possess a chiral symmetry in the DGO Standard Model, and so it made sense that it should make its appearance amid the charged leptons.

The nonuniform snub cube is also chiral, so if it is anywhere in the DGO SM, it should be as a chiral fermion of some kind (most likely a quark), although this remains to be seen.

We already know that GL(2,3) is linked to hyper-octahedral group and that, therefore, the quaternions and its associated rotations are also linked to it. If the octahedral group represented by Figure 1 can be associated with all 48 particles of the DGO SM, then this truncated CO is an object like the 'Philosopher's Stone'.

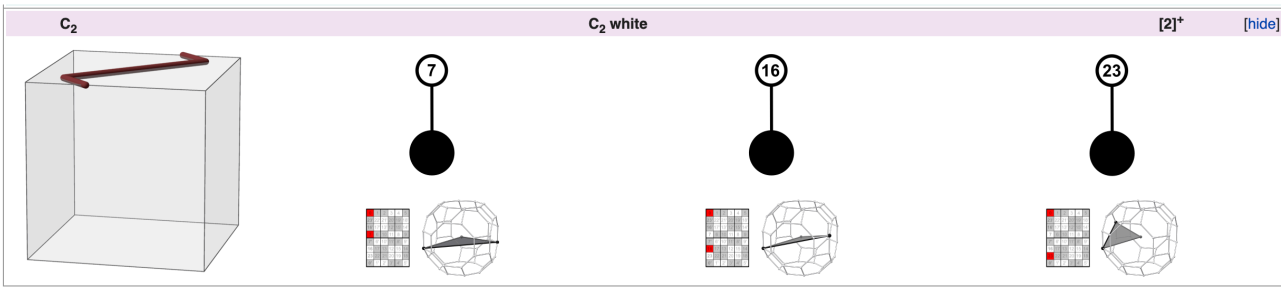

Figure 3: C_2 White Subgroup order 2 [1, 7]

Once we have the exact form of this pen-ultimate object, we may be able to use the already extremely advanced classifications of group theory to potentially deepen our understanding of which groups these particles belong to.

This is not too dissimilar to the proposal put forth in [6], wherein the rhombic dodecahedron was chosen to represent a set of all particles. That set was divided into subsets, numbering 6 and 8 particles and were simply added together to get 14, as opposed to multiplied in this case. This time, we are less interested in interactions and more interested in the particles and the groups they form and whether those groups can tell us anything new.

y	u	s	3	4	b
H-	22’	c*	20’	19’	18’
W-	17’	13’	12’	15’	t*
H+	d	10	11	8	9
H-	d*	10’	11’	8’	9’
W+	17	13	12	15	t
H+	22	c	20	19	18
Z	u*	s*	3’	4’	b*

Table 1: The quarks and bosons are added to the O_h symmetry table.

The truncated CO will contain the blueprint and model for all particle symmetry groups and subgroups. This could be very useful and interesting area of study.

But before we can do that we need to know how to place the particles on the appropriate vertices. This is the biggest challenge initially, as there is no absolute means of determining placements of particles and such a means may never exist. If you don’t know where to start; it is best to start small. So, I began with the non-trivial subgroups of order 2 and 3 and attempted to assign particles to their configurations.

For example, the C_2 subgroups appears to show the connection between the identity and 19 other particles, one of which must be its conjugate. Since it would appear that this identity must be present in all cases, it is assumed to be some kind of highly symmetric and ubiquitous particle (like the Higgs, the Graviton or the photon). The Higgs is a field which is everywhere but is only interacting with a limited style particle. The same is partially true of the Graviton. However, there is a theory that all matter and all particles may in fact be made of light. So let’s go ahead and use the photon (y) as our identity particle. Therefore, the other 19 particles are most likely;

{H+, H-, y, g, W^+, W^-, Z, u, d, s, c, t, b, u*, d*, s*, c*, t*, b*}

Notice that we've had to split the Higgs into a plus and minus charged particle. We can do this, because here are actually four Higgs particles according to the DGO. I began to fill in the S(4) based identifiers (See Table 1).

In this fashion, all of the particles can be transported onto the Stone. The process is similar to other attempts to unify the Standard Model with hyperdimensional objects like E_8. While the Stone does not appear to be multidimensional, at present that situation will change rapidly in subsequent preprints. Our main aim right now is to make new sets of particles and link them to a high-number of symmetry groups, which may be of some relevance to the spins of the particles in space, or for some other means of classification.

3. The Names of God

Once we have the correspondences in Table 1, we can check them against the rest of the O_h subgroups. We have to skip C_3, for now, but the Dihedral group 2, C_2, C_{2v}, and C_{2h} don't appear to cause any problems. C_4 and S_4 appear to conform to the charged and weakly interacting leptons, which means that C_3 was corresponding to the Gravitons. Indeed, it isn't until S_6, that we see all of the gravitons and gluons put into place. C_{2v} and C_{2h} covers the symmetry group of all of the meson arrangements.

0	1	2	3	4	5
23'	22'	21'	20'	19'	18'
16'	17'	13'	12'	15'	14'
7	6	10	11	8	9
7'	6'	10'	11'	8'	9'
16	17	13	12	15	14
23	22	21	20	19	18
0'	1'	2'	3'	4'	5'

(a)

y	u	s	G_1	G_2	b
H-	e*	c*	g_6	g_5	τ*
W-	ve*	vμ*	g_3	g_7	t*
H+	d	μ	G_4	G_8	vτ
H-	d*	μ*	g_4	g_8	vτ*
W+	ve	vμ	G_3	G_7	t
H+	e	c	G_6	G_5	τ
Z	u*	s*	g_1	g_2	b*

(b)

Table 2: (a) The Full Octahedral symmetry group. (b) A possible arrangement for the particles.

There are of course, other ways that we could have arranged this information. For example, the first column could have been arranged like this: {y, H-, H-, W+, W-, H+, H+, Z}. We also could have most likely switched the gluons with the gravitons. But the total number of variations is restricted by the logic of Oh symmetry groups and by the particles themselves. It is therefore not 48! or anything close to that number. An example of such a restriction lies in the association between anti-particles marked with '*' and reflections marked with an apostrophe. Other considerations come from grouping the particles according to the rules of the symmetry group itself. In the end, my reason for choosing {y, H-, W-, H+, H-, W+, H+, Z} was somewhat arbitrary and was ultimately informed by a consideration, which was neither scientific or mathematical. The keen-eyed will notice that the first four letters are synonymous with YHWH. This is something of a coincidence, but it would also appear to link back to another odd result containing the Tree of Life (see C4 in Table 4). We also notice that the word 'Zeu*s*' appears at the bottom of the table, indicating that the Ancient Greeks are likely vindicated in their implementation of comparative mythologies.

3 × rotation by 180° about a 4-fold axis	**inv2**	**7, 16, 23**	3 × reflection in a plane perpendicular to a 4-fold axis	**ref1**	**7', 16', 23'**
8 × rotation by 120° about a 3-fold axis	rot3	3, 4, 8, 11, 12, 15, 19, 20	8 × rotoreflection by 60°	rotref3	3', 4', 8', 11', 12', 15', 19', 20'
6 × rotation by 180° about a 2-fold axis	rot2	1', 2', 5', 6', 14', 21'	6 × reflection in a plane perpendicular to a 2-fold axis	ref2	1, 2, 5, 6, 14, 21
6 × rotation by 90° about a 4-fold axis	rot1	9', 10', 13', 17', 18', 22'	6 × rotoreflection by 90°	rotref1	9, 10, 13, 17, 18, 22

Table 3: Conjugacy classes [1]

A full list of these symmetry groups and how they correspond to the particles can be seen in Table 4 (or the 'Rocketship'). Some of these break down into still smaller groups, for example: C_2, and D_2. Interestingly, when grouped together like this are C_2, and D_2 are identical.

Order	**Subgroup**	**Particles**
2	**C_2**	y c* t* H+ d* W+ H+ u* s* b*
2	**C_s**	y u s b H- W- d t c
2	**S_2**	y Z

3	**C_3**	y G_1 G_2 G_4 G_8 G_6 G_5 G_3 G_7
4	**D_2**	y c* t* H+ d* W+ H+ u* s* b*
4	**C_{2v}**	y u s b H- W- H+ d y W+ t Z c
4	**C_{2h}**	y u s b H- c* W- t* H+ d H- d* W+ t H+ c Z u* s* b*
4	**C_4**	y e* τ* νe* νμ* H+ μ* ντ* W+ H+
4	**S_4**	y H- μ ντ W+ νe νμ H+ c τ
6	**S_6**	y G_1 G_2 g_6 g_5 g_3 g_7 G_4 G_8 g_1 g_2 G_6 G_5 G_3 G_7 Z g_4 g_8
6	**D_3**	y G_1 G_2 c* t* G_4 G_8 d* G_6 G_5 G_3 G_7 u* s* b*

6	**C_{3v}**	y u s G_1 G_2 b d G_4 G_8 G_6 G_5 t c G_3 G_7
8	**D_4**	y H- c* τ* W- vμ* t* H+ d* μ* vτ* W+ H+ u* s* b*
8	**D_{2d}**	y u s b H- τ* W- ve* vμ* t* H+ μ vτ H- d* W+ ve vμ t H+ e c τ u* s* b*
8	**C_{4v}**	y u s b H- e* τ* W- ve* vμ* t* H+ d H- μ* vτ* W+ t H+ c
8	**C_{4h}**	y e* τ* W- ve* H+ vτ H- vτ* W+ ve e τ Z
8	**D_{2h}**	y u s b H- c* W- t* H+ d H- d* W+ ve H+ c Z u* s* b*
12	**D_{3d}**	y u s G_1 G_2 b c* g_6 g_5 g_3 g_7 t* d G_4 G_8 d* g_1 g_2 G_6 G_5 t c G_3 G_7 Z u* s* g_4 g_8 b*
12	**T**	y G_1 G_2 H+ G_4 G_8 W+ G_6 G_5 H+ G_3 G_7

16	**D_{4h}**	y u H- e* W- ve* H+ d H- d* W+ ve H+ e Z u*
24	**T_h**	y G_1 G_2 H- g_6 g_5 W- g_3 g_7 H+ G_4 G_8 H- g_1 g_2 W+ G_6 G_5 H+ G_3 G_7 Z g_4 g_8
24	**O**	y G_1 G_2 e* c* τ* ve* vμ* t* H+ G_4 G_8 d* μ* vτ* W+ G_6 G_5 H+ G_3 G_7 u* s* b*
24	**T_d**	y u s G_1 G_2 b H+ d μ G_4 G_8 vτ W+ ve vμ G_6 G_5 t H+ e c G_3 G_7 τ
The group itself	**O_h**	y u s G_1 G_2 b H- e* c* g_6 g_5 τ* W- ve* vμ* g_3 g_7 t* H+ d μ G_4 G_8 vτ H- d* μ* g_1 g_2 vτ* W+ ve vμ G_6 G_5 t H+ e c G_3 G_7 τ Z u* s* g_4 g_8 b*

Table 4: The particles and the subgroups

We already noted the similarity between C_4 and the Tree of Life diagram. Necessarily, therefore, C_4 has 10 elements. But then we see that C_2, and D_2 also have ten elements. So there must be a link between the 2 groups of sets. Indeed, we see that C_4 deals with the anti-quarks and their mediating particles, while C_2, and D_2 cover the anti-leptons and the same group of bosons. Elsewhere, T_h is simply the set of all bosons and D_{4h} looks to be the set from which it is possible to derive the processes of Beta Decay.

Using these sets it should be possible to turn any particle into any other particle by a simple matter of rotation/reflection. But it is important to remember that these sets are not governing particle interactions. Instead, I conjecture that these sets represent mutual relationships between the particles, how they are formed or co-created by one another. Our table is also useful for error correction codes and shows a distinct although inverse relationship between the photon and the Z-boson, which would be interesting to explore further.

Those interested in the Tree of Life, it may be worth noting that the new group of particles are different from our last set [8]. Using the Octrahedral group we can also numerically reassign the Sephiroth in a number of ways. Using index order starting from '1', Chokhmah, Binah, Chesed, Gevurah, Tipheret, Netzach, Hod, Yesod and Malkuth equal {1, 8, 12, 14, 15, 19, 27, 30, 31, 37} respectively and {0, 22, 18, 17, 13, 7, 10, 9, 16, 23} using the absolute values of O_h.

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4. Cayley Tables

An alternative method to arranging the information in Table 2(b) is to use Cayley Tables. Half of the table for Oh is represented below (See Table 5). This is the automorphic group S4 and also the tetrahedral group (Td). [1]

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0	**y**	u	s	G1	G2	b	d	H+	G8	ντ	μ	G4	G3	νμ	t	G7	W+	ve	τ	G5	G6	c	e	H+
1	u	y	G2	b	s	G1	H+	d	μ	G4	G8	ντ	τ	G5	G6	c	e	H+	G3	νμ	t	G7	W+	ve
2	s	G1	y	u	b	G2	G3	νμ	t	G7	W+	ve	d	H+	G8	ντ	μ	G4	G5	τ	e	H+	G6	c
3	G1	s	b	G2	y	u	νμ	G3	W+	ve	t	G7	G5	τ	e	H+	G6	c	d	H+	G8	ντ	μ	G4
4	G2	b	u	y	G1	s	τ	G5	G6	c	e	H+	H+	d	μ	G4	G8	ντ	νμ	G3	W+	ve	t	G7
5	b	G2	G1	s	u	y	G5	τ	e	H+	G6	c	νμ	G3	W+	ve	t	G7	H+	d	μ	G4	G8	ντ
6	d	H+	G8	ντ	μ	G4	y	u	s	G1	G2	b	t	G7	G3	νμ	ve	W+	G6	c	τ	G5	H+	e
7	H+	d	μ	G4	G8	ντ	u	y	G2	b	s	G1	G6	c	τ	G5	H+	e	t	G7	G3	νμ	ve	W+
8	G8	ντ	d	H+	G4	μ	t	G7	G3	νμ	ve	W+	y	u	s	G1	G2	b	c	G6	H+	e	τ	G5
9	ντ	G8	G4	μ	d	H+	G7	t	ve	W+	G3	νμ	c	G6	H+	e	τ	G5	y	u	s	G1	G2	b
10	μ	G4	H+	d	ντ	G8	G6	c	τ	G5	H+	e	u	y	G2	b	s	G1	G7	t	ve	W+	G3	νμ
11	G4	μ	ντ	G8	H+	d	c	G6	H+	e	τ	G5	G7	t	ve	W+	G3	νμ	u	y	G2	b	s	G1
12	G3	νμ	t	G7	W+	ve	s	G1	y	u	b	G2	G8	ντ	d	H+	G4	μ	e	H+	G5	τ	c	G6
13	νμ	G3	W+	ve	t	G7	G1	s	b	G2	y	u	e	H+	G5	τ	c	G6	G8	ντ	d	H+	G4	μ
14	t	G7	G3	νμ	ve	W+	G8	ντ	d	H+	G4	μ	s	G1	y	u	b	G2	H+	e	c	G6	G5	τ
15	G7	t	ve	W+	G3	νμ	ντ	G8	G4	μ	d	H+	H+	e	c	G6	G5	τ	s	G1	y	u	b	G2
16	W+	ve	νμ	G3	G7	t	e	H+	G5	τ	c	G6	G1	s	b	G2	y	u	ντ	G8	G4	μ	d	H+
17	ve	W+	G7	t	νμ	G3	H+	e	c	G6	G5	τ	ντ	G8	G4	μ	d	H+	G1	s	b	G2	y	u
18	τ	G5	G6	c	e	H+	G2	b	u	y	G1	s	μ	G4	H+	d	ντ	G8	W+	ve	νμ	G3	G7	t
19	G5	τ	e	H+	G6	c	b	G2	G1	s	u	y	W+	ve	νμ	G3	G7	t	μ	G4	H+	d	ντ	G8
20	G6	c	τ	G5	H+	e	μ	G4	H+	d	ντ	G8	G2	b	u	y	G1	s	ve	W+	G7	t	νμ	G3
21	c	G6	H+	e	τ	G5	G4	μ	ντ	G8	7	d	ve	W+	G7	t	νμ	G3	G2	b	u	y	G1	s
22	e	H+	G5	τ	c	G6	W+	ve	νμ	G3	G7	t	b	G2	G1	s	u	y	G4	μ	ντ	G8	H+	d
23	H+	e	c	G6	G5	τ	ve	W+	G7	t	νμ	G3	G4	μ	ντ	G8	H+	d	b	G2	G1	s	u	y

Table 5: Half of the Cayley table for Oh using DGO particles. Notice that the quarks are blue, the leptons green and the bosons are white.

What is being demonstrated in Table 5 are the results of various rotations and reflections which lead to another particle. So, for example, t + t = y, whereas μ + μ = H+. Notice that we can also have rotations and reflections which lead to jumps between particles which are generally thought not to interact. For example, ve + G5 = t, according to one calculation on the above table, but there are equally many others, including vμ + G7 = e; and vτ + d = G1. These results are consistent because vτ - G1 = d. But then again, vτ + G6 = d also, so something a bit more complex is going on here. Something that is worth investigating further, in future articles and papers on the subject of Octahedral Symmetries, the DGO.

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